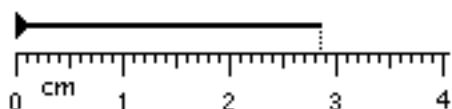


Measurements and Calculations Handout

A. Uncertainty in Measurement

Because people can only measure something to a certain degree of accuracy, it is important to realize that a measurement always has some degree of uncertainty. When one is making a measurement, the custom is to record all certain numbers plus the first uncertain number. The numbers recorded in a measurement (all the certain numbers plus the first uncertain number) are called significant figures.

1. Example in measuring length:



In this example the student records a measurement of 2.85 cm. The “2” and the “8” are certain numbers and the “5” is uncertain. The “5” is the student’s best guess. In other words, not every observer will necessarily record a “5”. Even so, the uncertain last number counts as significant. Therefore, this measurement has 3 significant figures.

B. Significant Figures

I. Rules for Counting Significant Figures:

1. *Nonzero digits.* Nonzero digits *always* count as significant figures. For example, the number 1457 has four nonzero digits, all of which count as significant figures.

2. *Zeros*

a. *Leading zeros* are zeros that *precede* all of the nonzero digits. They *never* count as significant figures. For example, in the number 0.0025, all three zeros indicate the position of the decimal point. The number has only two significant figures, the 2 and the 5.

b. *Captive zeros* are zeros that fall *between* nonzero digits. They *always* count as significant figures. For example, the number 1.008 has four significant digits.

c. *Trailing zeros* are zeros at the *right end* of the number, but not after a decimal point. They are significant only if the number contains a decimal point. For example, if the number one hundred is written 100, it only has 1 significant figure. However, if it is written as 100., then it has 3 significant figures.

d. *Super accuracy zeros* are zeros that fall to the right of a number after the decimal point. They are always significant, as are any zeros they hold captive to their left. For example, 1.00 has 3 significant figures, and 120.0 has 4 significant zeros.

3. **Exact numbers.** Often calculations involve numbers that were not obtained using measuring devices but were determined by counting: 10 experiments, 3 apples, 8 molecules. Such numbers are called *exact numbers*. They can be assumed to have an unlimited number of significant figures. Exact numbers can also arise from definitions. For example, 1 inch is defined as *exactly* 2.54 centimeters. Thus in the statement 1 inch = 2.54 centimeters, neither 2.54 nor 1 limits the number of significant figures when it is used in a calculation.

4. **Exponents never count as being significant.** For example, 8.26×10^{11} has three significant digits (the 8, the 2, and the 6). That's all.

II. Rules for Rounding Off

1. If the digit to be removed

- is less than 5, the preceding digit stays the same. For example, 1.33 rounds to 1.3.
- is equal to or greater than 5, the preceding digit is increased by one. For example, 1.36 rounds to 1.4, and 3.115 rounds to 3.12.

2. In a series of calculations, carry the extra digits through to the final result and then round off. This means that you should carry all the digits that show on your calculator until you arrive at the final number (the answer) and then round off, using the procedures in rule 1. **Never round off in the middle of a calculation!**

III. Rules for Using Significant Figures in Calculations

1. For **addition or subtraction**, the limiting term is the one with the smallest number of decimal places. We say this measurement is limiting, because it limits the accuracy of the result. For example,

12.11	←	2 decimal places
18.0	←	1 decimal place (limiting)
+ 1.013	←	3 decimals places
31.123		

↓
Round off

31.1	←	1 decimal place
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2. For *multiplication* or *division*, the number of significant figures in the result is the same as that in the measurement with the smallest number of significant figures. Here we say this measurement is limiting, because it limits the number of significant figures in the result. For example,



The product is correctly written as 6.4, which has two significant figures. Because 1.4 has only two significant figures, it limits the result to two significant figures.

Consider another example. In the division $8315 \div 298$, how many significant digits should appear in the answer?

Because 8315 has four significant figures, the number 298 (with three significant digits) limits the result. The calculation is correctly represented as



C. Scientific Notation

1. A number written in scientific notation consists of a **coefficient** (the part before the times sign) and an **exponent** (the power of 10 by which the coefficient is multiplied.) For example, in 4.3×10^6 (which equals 4,300,000, or four million three hundred thousand), 4.3 is the coefficient and 6 is the exponent. Sometimes the “times” symbol “x” is replaced by a dot, for example $4.3 \cdot 10^6$. In 2.3×10^{-8} (which equals .000000023), 2.3 is the coefficient and -8 is the exponent.

2. When you *multiply* two numbers, you *multiply* the coefficients and *add* the exponents. For example,

$$(4.3 \times 10^6) \times (2.0 \times 10^2) = 8.6 \times 10^8$$

$$(4.3 \times 10^6) \times (2.0 \times 10^{-2}) = 8.6 \times 10^4$$

3. When you *divide* two numbers, you *divide* the coefficients and *subtract* the exponents. For example,

$$(4.30 \times 10^6) \div (2.00 \times 10^2) = 2.15 \times 10^4$$

$$(4.30 \times 10^6) \div (2.00 \times 10^{-2}) = 2.15 \times 10^8$$

4. When you move the decimal place in the coefficient one position to the *left*, you *add* one to the exponent. For example,

$$42 \times 10^6 = 4.2 \times 10^7$$

$$4200 \times 10^6 = 4.2 \times 10^9$$

$$42 \times 10^{-6} = 4.2 \times 10^{-5}$$

5. When you move the decimal place in the coefficient one position to the *right*, you *subtract* one from the exponent. For example:

$$0.42 \times 10^6 = 4.2 \times 10^5$$

$$0.000043 \times 10^6 = 4.3 \times 10^1$$

$$0.42 \times 10^{-6} = 4.2 \times 10^{-7}$$

Note: You should always adjust the decimal place in the coefficient so that the coefficient is greater than one but less than ten. This is called standard form. Mathematically it doesn't make any difference, but that is the standard practice, and it does make a number easier to read.

6. When you *add* two exponents, you need to *make their exponents equal*. Take the number with the smaller exponent and move the decimal point to the left until its exponent matches the larger. Then *add* the coefficients and *keep* the (matching) exponent. Note that you might have to adjust the exponent when you are done to get into "standard form". For example:

$$(4.20 \times 10^6) + (6.40 \times 10^5) = (4.20 \times 10^6) + (0.640 \times 10^6) = 4.84 \times 10^6$$

$$(4.20 \times 10^{-6}) + (6.40 \times 10^{-5}) = (4.20 \times 10^{-5}) + (6.40 \times 10^{-5}) = 6.82 \times 10^{-5}$$

$$(9.20 \times 10^{11}) + (9.40 \times 10^{10}) = (9.20 \times 10^{11}) + (0.940 \times 10^{11}) = 10.14 \times 10^{11} = 1.014 \times 10^{12}$$

7. When you *subtract* two numbers, you again need to *make their exponents equal*. Take the number with the smaller exponent and move the decimal point to the left until its exponent matches the larger. Then *subtract* the coefficients and *keep* the (matching) exponent. For example,

$$(4.20 \times 10^6) - (6.40 \times 10^5) = (4.20 \times 10^6) - (0.640 \times 10^6) = 3.56 \times 10^6$$

$$(4.20 \times 10^{-6}) - (6.40 \times 10^{-5}) = (0.420 \times 10^{-5}) - (6.40 \times 10^{-5}) = -6.38 \times 10^{-5}$$

$$(9.20 \times 10^{11}) - (9.40 \times 10^{10}) = (9.20 \times 10^{11}) - (0.940 \times 10^{11}) = 8.26 \times 10^{11}$$

8. Using Calculators: Many calculators handle scientific notation. The exponent is usually displayed all the way to the right, with a space between it and the coefficient. To enter a number in scientific notation you enter the coefficient, press the EXP key (on some calculators it is labeled EE) and enter the exponent. To enter the number 130000000, you have to enter 1.3 EXP 8. See your calculator instruction booklet for more help. Make sure you understand your calculator *before* you count on it to get you through a test!